limits when x approaches infinity

By: Meshari Alsulaiteen, Abdulaziz Al Jenae

Content

1 Polynomial

Example:

$$\lim_{x\to-\infty} 3x^4 - 2x + \boxed{x^5} = \lim_{x\to-\infty} x^5 = (-\infty)^5 = -\infty$$

Solve:

1) $f(x) = x^3 + 2x^4 + \frac{3}{7}x^7$

Rational 2

Example:

$$= 0 + 1 + 3(\infty) = \infty$$

- $$\lim_{x\to-\infty} \frac{3x^{-3}+1+x}{2x} = \lim_{x\to-\infty} \frac{\overset{0}{\frac{3}{x^3}}+1+x}{2x} = \frac{-\infty}{-\infty} \text{ I.F.}$$

$$= \lim_{x\to-\infty} \frac{\frac{3+x^3+x^4}{x^3}}{2x} = \lim_{x\to-\infty} \frac{3+x^3+\boxed{x^4}}{2\boxed{x^4}}$$

$$= \lim_{x\to-\infty} \frac{x^4}{2x^4} = \frac{1}{2}$$

Solve:

$$\lim_{x\to\infty} \frac{-3x^5+2x-5}{x^2}$$

3 Radical

Example:

$$\lim_{x\to-\infty}\left(\sqrt{x^2+6x}+x\right) = \lim_{x\to-\infty} \frac{\left(\sqrt{x^2+6x}+x\right)\left(\sqrt{x^2+6x}-x\right)}{\sqrt{x^2+6x}-x}$$ conjugates.

$$= \lim_{x\to-\infty} \frac{\cancel{x^2}+6x-\cancel{x^2}}{\sqrt{x^2+6x}-x} = \lim_{x\to-\infty} \frac{6x}{\sqrt{x^2\left(1+\frac{6}{x}\right)}-x} = \lim_{x\to-\infty} \frac{6x}{|x|\sqrt{1+\frac{6}{x}}-x}$$

$$= \lim_{x\to-\infty} \frac{6x}{-x\sqrt{1+\frac{6}{x}}-x} = \lim_{x\to-\infty} \frac{6\cancel{x}}{-\cancel{x}\left(\sqrt{1+\frac{6}{x}}+1\right)}$$

$$= \frac{6}{-(\sqrt{1}+1)} = \frac{-6}{2} = -3$$

$y=-3$ is a HA

Solve:

$$\lim_{x\to+\infty}\left(\sqrt{a^2x^2+x}-ax\right)$$

4

combination of radical and rational

Example:

$$\lim_{x\to-\infty}\frac{\sqrt{2x^2+1}}{4x+2} = \lim_{x\to-\infty}\frac{\sqrt{x^2\left(2+\frac{1}{x^2}\right)}}{4x+2} = \lim_{x\to-\infty}\frac{-\cancel{x}\sqrt{2+\frac{1}{x^2}}}{\cancel{x}\left(4+\frac{2}{x}\right)} = -\frac{\sqrt{2}}{4}$$

$y = -\frac{\sqrt{2}}{4}$ is HA

Solve:

4) $f(x) = \dfrac{\sqrt{2x^2+7x}}{2x+1}$

5

Exponential function

Example:

$$\lim_{x\to-\infty} \frac{2^{2x+3}}{5^{x-1}} = \lim_{x\to-\infty} \frac{2^{2x}\cdot 2^3}{\frac{5^x}{5}} = \lim_{x\to-\infty} 2^{2x}\cdot 2^3 \cdot \frac{5}{5^x} = \lim_{x\to-\infty} \frac{4^x}{5^x}\cdot 40$$

$$= \lim_{x\to-\infty} \underbrace{\left(\frac{4}{5}\right)^x}_{\infty} \cdot 40 \qquad = 40(\infty) = \infty$$

Solve:

7.) $\displaystyle \lim_{x\to\infty} \frac{2^{2x+1}}{3^{x-1}}$

Natural exponential

6

Formula:

Formulas needed:

- $\lim\limits_{x \to -\infty} e^{x} = 0$ means $e^{-\infty} = 0$
- $\lim\limits_{x \to +\infty} e^{x} = \infty$ means $e^{\infty} = \infty$
- $\dfrac{e^{a}}{e^{b}} = e^{a-b}$
- $e^{a} \cdot e^{b} = e^{a+b}$

Example:

$$\lim_{x \to \infty} e^{2-4x-8x^2} = \lim_{x \to \infty} e^{-8x^2} = \lim_{x \to \infty} e^{-8(\infty)^2} = e^{-8(\infty)} = e^{-\infty} = 0$$

Solve:

1. $f(x) = e^{8+2x-x^3}$

Example:

(b) $\lim_{x\to-\infty} \left(\frac{e^{10x} - 4e^{6x} + 3e^{x} + 2e^{-2x} - 9e^{-15x}}{e^{-15x}}\right)$

$\frac{e^a}{e^b} = e^{a-b}$

take e with the highest (-) superscript $(-15x)$

$$= \lim_{x\to-\infty} e^{-15x}\left(\frac{e^{10x}}{e^{-15x}} - 4\frac{e^{6x}}{e^{-15x}} + \frac{3e^{x}}{e^{-15x}} + \frac{2e^{-2x}}{e^{-15x}} - 9\right)$$

$$= \lim_{x\to-\infty} e^{-15x}\left(\underbrace{e^{\overset{-\infty}{25x}}}_{0} - 4\underbrace{e^{\overset{-\infty}{21x}}}_{0} + 3\underbrace{e^{\overset{-\infty}{16x}}}_{0} + 2\underbrace{e^{\overset{-\infty}{13x}}}_{0} - 9\right)$$

$$= e^{-15(-\infty)}(-9) = e^{\infty}\cdot(-9) = \infty(-9) = -\infty$$

Solve:

8. $f(x) = 20e^{-8x} - e^{5x} + 3e^{2x} - e^{-7x}$

Example:

bre 2020 10:51

$$\lim_{x\to+\infty} e^{\left(\frac{3x^2+2x+2}{2x^2+x^{-3}+4}\right)} = \lim_{x\to+\infty} e^{\left(\frac{3x^2+2x+2}{2x^2+\frac{1}{x^3}+4}\right)}$$

$$= \lim_{x\to+\infty} e^{\left(\frac{3x^2+2x+2}{\frac{2x^5+1+4x^3}{x^3}}\right)}$$

$$= \lim_{x\to+\infty} e^{\left(\frac{3x^5+2x^4+2x^3}{2x^5+1+4x^3}\right)}$$

$$= \lim_{x\to+\infty} e^{\frac{3}{2}} = e^{3/2}$$

Solve:

5. $f(x) = \mathbf{e}^{\frac{5+2x^6}{x-8x^4}}$

7 Natural logarithmic function

Formula:

Prop:

Expand

1. $\ln(a \cdot b) = \ln(a) + \ln(b)$ Exp: $\ln(2x) = \ln(2) + \ln(x)$

simplify

2. $\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$ Exp: $\ln\left(\frac{2x}{x^3}\right) = \ln(2x) - \ln(x^3)$

3. $\ln x^n = n \ln x$ Exp: $\ln x^3 = 3 \ln x$

Example:

$$\lim_{x \to +\infty} \ln(3x^2 + 2x + 1)$$

$3(\infty)^2 = \infty$

when x approches $+\infty$; $3x^2 + 2x + 1 \longrightarrow +\infty$

. let $u = 3x^2 + 2x + 1$: $u \longrightarrow +\infty$

Solve:

7. $\lim\limits_{t \to -\infty} \ln\left(4 - 9t - t^3\right)$

Example:

$$\lim_{x \to +\infty} \operatorname{Ln}\left(\frac{3x^2+4}{x^5-1}\right)$$

$\frac{\infty}{\infty}$ IF $\lim_{x \to +\infty} \frac{3x^2}{x^5} = \lim_{x \to +\infty} \frac{3}{x^3}$

$= \frac{3}{\infty} = 0.$

- $x \to +\infty$; $\frac{\boxed{3x^2}+4}{\boxed{x^5}-1} \longrightarrow \underline{\underline{0}}$

- let $u = \frac{3x^2+4}{x^5-1}$; $u \longrightarrow 0$

$$\lim_{x \to +\infty} \operatorname{Ln}\left(\frac{3x^2+4}{x^5-1}\right) = \lim_{u \to 0} \operatorname{Ln}(u) = -\infty.$$

Solve:

8. $\lim\limits_{z \to -\infty} \ln\left(\frac{3z^4-8}{2+z^2}\right)$

Example:

$$\lim_{x\to\infty} \frac{\ln\left(\frac{3}{x^3}\right)}{3 - \ln\left(\frac{2}{x}\right)} = \lim_{x\to\infty} \frac{\ln(3) - \ln x^3}{3 - \ln(2) + \ln x}$$

$$3 - (\ln(2) - \ln(x))$$

$$3 - \ln(2) + \ln x$$

$$= \lim_{x\to\infty} \frac{\ln(3) - 3\ln x}{3 - \ln(2) + \ln x}$$

$$= \lim_{x\to\infty} \frac{\underbrace{\frac{\ln(3)}{\ln x}}_{0} - \frac{3\cancel{\ln x}}{\cancel{\ln x}}}{\underbrace{\frac{3}{\ln x}}_{0} - \underbrace{\frac{\ln(2)}{\ln x}}_{0} + \underbrace{\frac{\cancel{\ln x}}{\cancel{\ln x}}}_{1}} = \frac{-3}{1} = -3$$

Solve:

$$\lim_{x\to\infty} \left(\frac{\ln \frac{1}{x^2}}{1 + \ln \frac{3}{x}} \right)$$

8

Relationship between limits and asymptotes

Limits at Infinity and Horizontal Asymptotes

From earlier work, you know that $y = \frac{1}{2}$ is a horizontal asymptote of the graph of this function.

Using limit notation, this can be written as follows.

$$\lim_{x \to -\infty} f(x) = \frac{1}{2}$$ Horizontal asymptote to the left

$$\lim_{x \to \infty} f(x) = \frac{1}{2}$$ Horizontal asymptote to the right

These limits mean that the value of $f(x)$ gets arbitrarily close to $\frac{1}{2}$ as x decreases or increases without bound.

Limits and Asymptotes

$x = a$ is a vertical asymptote of $f(x)$ if at least one of the following is true.

$$\lim_{x \to a} f(x) = \infty \qquad \lim_{x \to a} f(x) = -\infty$$

$$\lim_{x \to a^+} f(x) = \infty \qquad \lim_{x \to a^+} f(x) = -\infty$$

$$\lim_{x \to a^-} f(x) = \infty \qquad \lim_{x \to a^-} f(x) = -\infty$$

$y = L$ is a horizontal asymptote of $y = f(x)$ if either of the following is true.

$$\lim_{x \to \infty} f(x) = L \qquad \lim_{x \to -\infty} f(x) = L$$

Thank You